

# Simulator Investigation of the Effects of $L_\alpha$ and True Speed on Longitudinal Handling Qualities

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The experimental study conducted was directed toward investigating the effects, on pilot rating and optimum longitudinal control gain, of large variations in the relative amplitude and phase of the basic airplane responses to elevator control and to vertical gusts. The variations in relative amplitude and phase were obtained through changes in true speed and the parameter  $L_\alpha = (\rho SV/2m)C_{L\alpha}$ . Combinations of  $V$  and  $L_\alpha$  were examined for each of three short period poles in a fixed-base simulator. True speed was varied from  $V = 130$  knots to  $V = 1800$  knots and  $L_\alpha$  was varied from  $L_\alpha = 0.07$  1/sec to  $4.01$  1/sec. The steady state ratio of normal acceleration to angle of attack was found to be of significance both to the flying qualities of an airplane and to the optimum longitudinal control gain. Thus the results of previous investigations of the effect of short period dynamics on longitudinal flying qualities must be qualified since the investigations were conducted at specific values of  $L_\alpha$  and  $V$  or  $n_z/\alpha$ . The longitudinal control gain selected by the pilots was found to be a compromise which weighed the often conflicting requirements for good pitch attitude control, adequate  $g$ -limiting protection and satisfactory steady forces in turns. It was demonstrated that the primary factors which determine the normal acceleration response to turbulence are  $L_\alpha$ , short period frequency, short period damping ratio, and true speed together with the frequency spectrum characteristics of the turbulence.

## Nomenclature

$h$	= altitude
$\rho$	= air density
$q_0$	= dynamic pressure, $= \frac{1}{2}\rho V^2$
$S$	= wing area
$c$	= wing chord
$m$	= mass
$W$	= weight
$g$	= acceleration of gravity, $= 32.2$ fps <sup>2</sup>
$I_{yy}$	= airplane moment of inertia about body axis
$V$	= airspeed
$u$	= incremental airspeed
$w_{ag}$	= vertical gust velocity, positive up
$\alpha$	= angle of attack
$\gamma$	= flight path angle, positive up
$\delta_e$	= elevator angle, positive for trailing edge down
$\delta_{ES}$	= elevator stick deflection, positive back
$X, Y, Z$	= reference axes
$\theta$	= attitude angle, angle between $X$ axis and horizontal plane
$a_z$	= component of acceleration of airplane c.g. along $Z$ axis
$n_z$	= normal accelerometer reading in $g$ units, positive in pullup
$F_s$	= stick force
$L$	= lift, force in plane of symmetry and normal to relative wind, positive up
$L_\alpha$	$= (1/m)(\partial L/\partial \alpha)$ , where $\partial L/\partial \alpha = \lim_{\Delta \alpha \rightarrow 0} \Delta L/\Delta \alpha$ $L_\alpha \cong -Z_w$ in Ref. 1.
$X$	= aerodynamic force along $X$ wind axis, positive forward
$Z$	= aerodynamic force along $Z$ wind axis, positive down
$M$	= pitching moment about $Y$ axis, positive noseup
$C_D$	= drag coefficient $= D/q_0 S$
$C_L$	= lift coefficient $= L/q_0 S$
$C_m$	= pitching moment coefficient $= M/q_0 S c$

The following dimensional units are used: distance, ft; time, sec; angle, radians; force, lb; moment, ft-lb; and mass, slugs.

The following stability derivative notation is used:

$$C_{D\alpha} = \frac{\partial C_D}{\partial \alpha} \quad C_{m\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{L\alpha} = \frac{\partial C_L}{\partial \alpha} \quad C_{m\delta_e} = \frac{\partial C_m}{\partial \delta_e}$$

$$C_{L\delta_e} = \frac{\partial C_L}{\partial \delta_e} \quad C_{m\dot{\alpha}} = \frac{2V}{c} \frac{\partial C_m}{\partial \dot{\alpha}}$$

$$C_{mq} = \frac{2V}{c} \frac{\partial C_m}{\partial q}$$

The following dimensional stability derivative notation is used:

$$-Z_w \cong L_\alpha = \frac{\rho SV}{2m} C_{L\alpha} \quad L_{\dot{\delta}} = \frac{\rho SV}{2m} C_{L\delta_e}$$

$$M_\alpha = \frac{q_0 S c}{I_{yy}} C_{m\alpha} \quad M_{\dot{\delta}} = \frac{q_0 S c}{I_{yy}} C_{m\delta_e}$$

$$M_{\dot{\alpha}} = \frac{q_0 S c}{I_{yy}} \frac{c}{2V} C_{m\dot{\alpha}} \quad M_q = \frac{q_0 S c}{I_{yy}} \frac{c}{2V} C_{mq}$$

$s$	= Laplace operator
$\omega$	= frequency, rad/sec
$\omega_n$	= undamped natural frequency, rad/sec
$f_n$	= undamped natural frequency, cps
$\zeta$	= damping ratio
$\kappa$	= gain factor
$\tau$	= time constant
$C_n$	= white noise power spectra intensity
$\phi(\omega)$	= power spectral density
$G(j\omega)$	= filter transfer function
$Y(j\omega)$	= airplane transfer function

## Introduction

IN the past considerable handling quality research has been focused on the aircraft's natural modes of motion which are described by the characteristic equation or transfer function denominators. Although the characteristic equation alone defines the frequency and damping ratio or the time constants of the natural modes of motion, the numerator terms of the airplane transfer functions contribute to the

Presented at the AIAA Simulation for Aerospace Flight Conference, Columbus, Ohio, August 26-28, 1963 (no preprint number; published in bound volume of preprints of the meeting); revision received August 28, 1964. This paper is based on the results of an investigation performed under Contract AF33(657)-7442 for the Flight Control Laboratory, Aeronautical Systems Division, United States Air Force.

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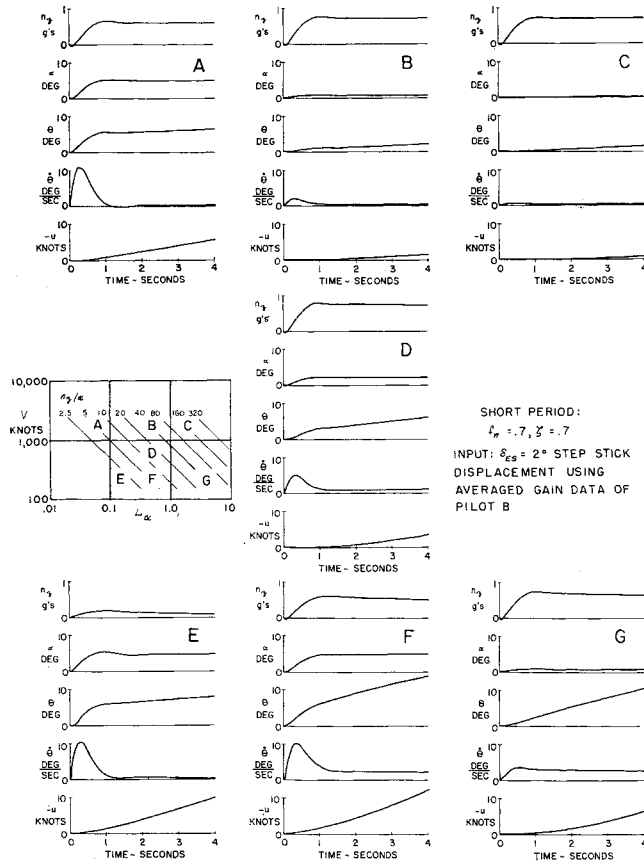


Fig. 1 Longitudinal response to 2° elevator stick step input.

relative amplitude and phase of the various airplane responses and thus are significant also in determination of handling qualities.

Although limited six-degree-of-freedom equations were used in the simulation program, the following simplified longitudinal equations of motion illustrate the major points to be discussed:

$$\ddot{\theta} = M_q \dot{\theta} + M_{\dot{\alpha}} \dot{\alpha} + M_{\alpha} \alpha + M_{\delta} \delta_e \quad (1)$$

$$\dot{\alpha} = \dot{\theta} - L_{\alpha} \alpha \quad (2)$$

Also

$$\dot{\gamma} = \dot{\theta} - \dot{\alpha} \quad (3)$$

$$n_z = -(V/g) \dot{\gamma} \quad (4)$$

These equations assume wings-level, constant speed flight. Also, they neglect lift due to elevator deflection and the incremental effects of gravity.

From these equations the following simplified transfer functions can be developed:

$$\frac{\dot{\theta}(s)}{\delta_e(s)} = K_{\dot{\theta}} \frac{(\tau \dot{\theta} s + 1)}{(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1} \quad (5)$$

$$\frac{\alpha(s)}{\delta_e(s)} = K_{\alpha} \frac{1}{(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1} \quad (6)$$

$$\frac{n_z(s)}{\delta_e(s)} = K_{n_z} \frac{1}{(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1} \quad (7)$$

where

$$\begin{aligned} \omega_n^2 &= -M_{\alpha} - M_q L_{\alpha} & K_{\alpha} &= M_{\delta}/\omega_n^2 \\ \zeta &= (L_{\alpha} - M_q - M_{\dot{\alpha}})/2\omega_n & K_{\dot{\theta}} &= L_{\alpha}(M_{\delta}/\omega_n^2) \\ \dot{\theta} &= 1/L_{\alpha} & K_{n_z} &= (VL_{\alpha}/g)(M_{\delta}/\omega_n^2) \end{aligned}$$

Using these simplified transfer functions, the relative amplitude and phase of any two of the various airplane responses to elevator control can be obtained by taking the ratio of the particular transfer functions for elevator input:

$$\dot{\theta}(s)/n_z(s) = (g/V)[(1/L_{\alpha})s + 1] \quad (8)$$

$$\dot{\theta}(s)/\alpha(s) = L_{\alpha}[(1/L_{\alpha})s + 1] \quad (9)$$

$$\frac{\gamma(s)}{\theta(s)} = \frac{1}{[(1/L_{\alpha})s + 1]} \quad (10)$$

$$n_z(s)/\alpha(s) = L_{\alpha}V/g \quad (11)$$

$$\dot{\gamma}(s)/\alpha(s) = L_{\alpha} \quad (12)$$

where

$$L_{\alpha} = q_0 SC L_{\alpha}/mV = [g\rho V/2(W/S)]C_{L_{\alpha}} \quad (13)$$

From these expressions, it is seen that the relative magnitudes and phases of the aircraft responses are primarily determined by the true speed, the gravitational constant, and the parameter  $L_{\alpha}$ , where  $L_{\alpha}$  is the coefficient of the angle of attack term in the lift equation.  $L_{\alpha}$  is proportional to  $g$ , air density, true speed, and lift curve slope, and inversely proportional to wing loading.

Note that the pitch rate leads the normal acceleration and angle of attack, and that the flight path lags the pitch attitude. The lead and lag time constants are inversely proportional to  $L_{\alpha}$ . Also, the steady state normal acceleration is related to steady state pitch rate by  $V/g$  and to angle of attack by  $L_{\alpha}V/g$ . The steady state pitch rate and rate of change of flight path angle are related to angle of attack by  $L_{\alpha}$ .

Time histories of normal acceleration, angle of attack, pitch attitude, pitch rate, and incremental velocity are plotted in Fig. 1 for seven combinations of  $L_{\alpha}$  and  $V$  for the short period pole  $f_n = 0.7$  cps and  $\zeta = 0.7$ . The individual and combined effects of  $L_{\alpha}$  and  $V$  on the relative magnitude and phase of the vehicle responses can be seen through detailed examination of the plotted responses of Fig. 1 together with the approximate expressions developed in Eqs. (8-12). These effects will be discussed in some detail in a following section.

The effects of  $L_{\alpha}$  and true speed have become of particular interest because of the greatly expanded flight envelope of current and future piloted vehicles. Various combinations of vehicle design, configuration, and flight condition can result in a wide range of values of both  $L_{\alpha}$  and true speed. For this reason a ground simulator program was planned and conducted to explore the effects of  $L_{\alpha}$  and true speed on pilot rating of longitudinal flying qualities and on optimum control gain at three short period poles.

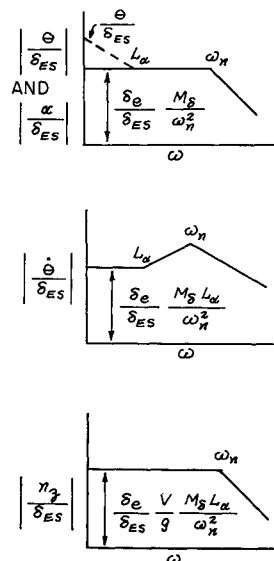


Fig. 2 Longitudinal control gains assuming constant speed and negligible elevator lift.

### Longitudinal Control Gains

The gain constants of the factored transfer functions developed in Eqs. (5-7) express the ratio of the magnitude of each airplane response to the magnitude of the elevator input. If these gains are multiplied by the gear ratio relating stick displacement to elevator displacement, the gains between pilot input and airplane responses are obtained. These gains are illustrated by the asymptotic Bode plots in Fig. 2. Assuming constant speed and negligible elevator lift, the steady state angle of attack gain is seen to be equal to the pitch attitude gain at the short period, the steady state pitch rate gain is  $L_\alpha$  times the angle of attack gain, and the steady state normal acceleration gain is  $V/gL_\alpha$  times the angle of attack gain. From these relationships it is seen that, depending on which response the pilot is controlling, the gearing may or may not need to be modified as velocity or  $L_\alpha$  change.

In the ground simulator program, the pilots were asked to select the stick to elevator gear ratio that they considered most compatible with each configuration evaluated. Thus, for example, if the pilot preferred constant pitch gain we would expect him to select a different value of  $\delta_e/\delta_{ES}M_\delta$  when  $\omega_n^2$  or  $L_\alpha$  were changed but not when the velocity was changed.

In this way, data were collected to determine the functional relations that might exist between the pilot selected values of the gains defined in Fig. 2 and the independent variables  $V$ ,  $L_\alpha$ , and  $\omega_n$ . The corresponding stick force characteristics will be discussed in a later section.

### Longitudinal Response to Turbulence

An important consideration in the evaluation of an aircraft from the handling qualities point of view is its response to atmospheric turbulence. The acceleration environment that the pilot and crew must endure is determined by the turbulence characteristics and by the response of the aircraft to the turbulence.

The details of the turbulence simulation used in the experiment are described in Ref. 2. The spectral distribution of the turbulence input used in the experiment is illustrated in Fig. 3. Under the assumptions of Ref. 2, the normal acceleration response of the uncontrolled airplane to vertical gust inputs is expressed in transfer function form as

$$\frac{n_z(s)}{w_{ag}(s)} = \frac{L_\alpha s[s - (M_q + M_{\dot{\alpha}})]}{g[s^2 + 2\zeta\omega_n s + \omega_n^2]} \quad (14)$$

The magnitude of the amplitude ratio of this transfer function as a function of frequency is illustrated by Fig. 4. From Eq. (14) and the sketch of Fig. 4, it is seen that, at very low frequency, the acceleration response is low, whereas at high frequency, the acceleration response is equal to  $L_\alpha/g$ . The response at intermediate frequencies is determined by the short period frequency and damping ratio, and by the numerator break point at  $\omega = M_q + M_{\dot{\alpha}}$ . Thus the normal acceleration response would be expected to be a minimum for a low value of  $L_\alpha$  and a high short period natural frequency.

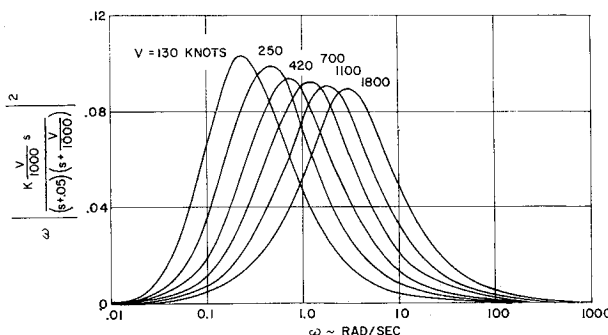


Fig. 3 Simulated turbulence spectra.

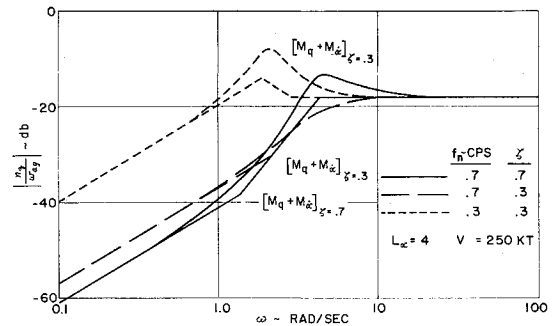


Fig. 4 Normal acceleration response to vertical gust.

The effect of short period damping is to reduce the response at the short period frequency and to increase the response at low frequency due to the larger value of  $(M_q + M_{\dot{\alpha}})$ ; i.e., for constant  $L_\alpha$  and  $\omega_n$ , the short period damping ratio can only be increased by increasing  $(M_q + M_{\dot{\alpha}})$ , thus increasing the response at low frequency.

The net normal acceleration frequency spectra can be obtained by combining the simulated turbulence spectra of Fig. 3 with the airplane transfer function, Eq. (14). This has been done for each of the three short period poles of Fig. 4 for  $L_\alpha = 4$  and  $V = 250$  knots. The resulting normal acceleration frequency spectra are plotted in Fig. 5. For this plot, the power spectra has been multiplied by  $\omega$  and plotted vs frequency on a log scale. The shape of the resulting curve illustrates at a glance which frequencies contribute most to the normal acceleration response. The area under each of the curves in Fig. 5 is proportional to the mean square normal acceleration response of that configuration. The three curves of Fig. 5 graphically illustrate the effect of short period frequency and damping ratio on the normal acceleration response to vertical gusts. The ground simulator program was designed in part to obtain systematic data on the effects and interactions of the above discussed factors on crew comfort and the handling qualities of an aircraft flying in turbulent air.

## Experimental Procedure

### Experiment Design

Handling quality evaluation tests were conducted in a fixed-base flight simulator for various combinations of  $L_\alpha$  and  $V$  at each of three short period pole locations. The numerical values of the various parameters studied for short period poles are  $f_n \sim 0.7, 0.7$ , and  $0.3$  cps and  $\zeta \sim 0.7, 0.3$ , and  $0.3$ . The location of these poles relative to the handling qualities boundaries established by variable stability airplanes is shown in Fig. 6. The true speed values are  $V \sim 130, 250, 420, 700, 1100$ , and  $1800$  knots. The  $L_\alpha$  values are  $L_\alpha \sim 0.07, 0.15, 0.50, 1.8$ , and  $4.0$  1/sec.

In conducting the tests the computer and instrument display were set up for a given velocity, and then each of the

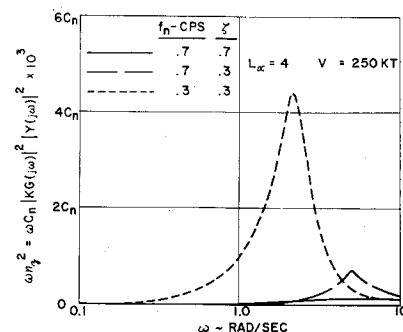


Fig. 5 Normal acceleration response to simulated turbulence.

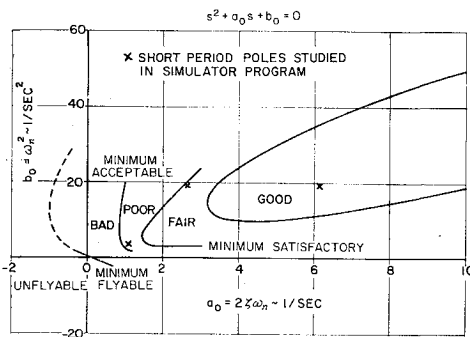


Fig. 6 Longitudinal short period dynamics (variable stability airplane data).

possible combinations of four values of  $L_\alpha$  and three short period poles was studied. For each such group of 12 configurations, one configuration was repeated three times. Thus, at each speed a set of 15 configurations was studied. An exception to this schedule was the  $V = 130$  knot speed for which only three values of  $L_\alpha$  were studied at one short period pole.

The order in which the configurations were studied was random or at least arbitrary. The pilots had no foreknowledge of the particular configurations that were presented to them. The following data were recorded for each configuration: 1) optimum elevator gear ratio; 2) pilot ratings; 3) pilot comment data; 4) aircraft response to elevator pulse input, elevator step input, throttle step input, and two-minute samples of pilot attempting to minimize disturbances to attitude, altitude, and airspeed in rough air.

Items 1, 2, and 3 form the bulk of the data and are analyzed in the following section. The oscillograph records of aircraft responses were used mainly to establish proper operation of the simulation equipment.

#### Description of the Simulator

The Air Force-Cornell variable stability T-33 was used in this program as a fixed-base simulator. It was parked in the corner of the Cornell flight hangar and connected through cabling to an analog computer. The pilot sat in the front cockpit, which was covered, and "flew" the airplane using a two-axis side stick, rudder pedals, and a simulated throttle. The side stick produced an electrical signal proportional to displacement with feel provided by springs and Teflon disk dampers. The rudder pedals were positioned by the pilot's feet and produced electrical signals proportional to displacement. The rudder pedal feel was provided by feel servos. These control signals were used to command the hydraulic servos which position the control surfaces. Position pickups on the control surfaces generated electrical signals which were

used together with a signal from the throttle as inputs to the analog computer. With this arrangement the dynamics of the control system were included as part of the simulation.

The computed airplane responses were used to drive the display instruments in the front cockpit of the T-33, thus closing the loop through the pilot. The active instruments in the display were airspeed, rate of climb, altitude, pitch and roll attitude, heading, angle of attack, normal acceleration, sideslip, and rate of turn. Reference 3 describes the T-33 simulation equipment.

#### Subjects in the Experiment

The three pilots who participated in this program are all members of the Flight Research Department of Cornell Aeronautical Laboratory, Inc. Two have engineering degrees and all have participated in various handling quality evaluation programs in the past.

#### Instructions to Pilots

The briefing instructions to the pilots who participated in the program were formalized in a memorandum and a set of "flight cards." These instructions established the frame of reference in which the evaluations were performed. The pilots were instructed to select the elevator gear ratio that they considered most compatible for each configuration and to conduct their evaluation using this "optimum" elevator control sensitivity. The evaluations were conducted in two parts and two ratings were assigned to each configuration. The first part of the evaluation was conducted in "smooth air," i.e., no external disturbances to the airplane. Upon completion of the smooth air evaluation and rating, the pilots examined the configuration in simulated rough air. This part of the evaluation was conducted to determine whether the rough air caused any particular difficulties in control and to determine the severity of the ride that might be expected for flight in continuous turbulence. The rating scale shown in Table 1 was used in the program.

This rating scale is similar to the Cooper rating scale,<sup>†</sup> but there is a difference to be noted in respect to its use and interpretation. In Cornell flying qualities research programs, this scale is used as a general scale for evaluating the suitability of a configuration for any single specific task or mission. The Cooper definitions introduce toward the bottom of the scale the elements of emergency operation and stability augmentation failure. This raises the question of whether the task is fixed. In going down the Cooper scale, does the task change from performing the primary mission to accomplishing a safe return home? If not, it seems at least indirect and possibly confusing to specify degree of suitability in terms of the supposed occurrence of failure or emergency conditions. The scale presented above, as used at Cornell, is based on drawing a clear distinction between definition of the task and definition of the degrees of suitability that constitute the rating scale. A further point is that the Cornell scale is a ten-point rating scale, whereas the Cooper scale is in practice a nine-point scale. This is so because Cooper rating no. 9 is "uncontrollable." The two scales probably correspond pretty well at the dividing line of 3.5. Cornell experience indicates that it is desirable to have the four ratings from 7-10 defined in usable terms.

#### Evaluation Criteria

The study was intended to be a general survey or exploration of the effect of pilot ratings and optimum longitudinal control sensitivity, of the large variations in the relative amplitude and phase of the basic airplane responses that occur when true speed and  $L_\alpha$  are changed. Because the study was a general one and the range of velocity and  $L_\alpha$  was so wide, it

Table 1 Ten point rating scale for airplane handling qualities

Category	Adjective description within category	Numerical rating
Acceptable and satisfactory	Excellent	1
	Good	2
	Fair	3
Acceptable but unsatisfactory	Fair	4
	Poor	5
	Bad	6
Unacceptable	Bad <sup>a</sup>	7
	Very Bad <sup>b</sup>	8
	Dangerous <sup>c</sup>	9
Unflyable	Unflyable	10

<sup>a</sup> Requires major portion of pilots' attention.

<sup>b</sup> Controllable only with a minimum of cockpit duties.

<sup>c</sup> Aircraft just controllable with complete attention.

<sup>†</sup> The Cooper rating scale is defined in Ref. 9.

was not possible to define a single vehicle or a specific mission for the evaluations. Therefore, the vehicle type was left undefined except that it was a powered lifting vehicle capable of the speed displayed to the pilot. The airplanes were assumed to be in "up and away" flight under instrument conditions and the evaluation task was made the general one of maintaining precise control of the flight path of the vehicle through space.

Thus, the smooth air ratings were based on the amount of effort the pilot was required to put forth to achieve the precision of flight path control that he considered satisfactory. The ratings were determined individually while each pilot actually performed the task, or at least the components of the task, and included his evaluation of the effort, skill, concentration, and the practicability of any special techniques required to accomplish the task, as well as, his performance in actually accomplishing it. The rating also reflected whether or not the configuration possessed any characteristic that the pilot considered to be potentially dangerous or unforgiving.

The rough air rating was intended to reflect any control difficulties caused by external disturbances and to indicate the severity of the ride that might be expected in continuous turbulence. All three pilots adopted the philosophy that their ability to control the flight path could never be better in rough air than it was in smooth air. Thus, the ratings after rough air were either the same as in smooth air or reflected a degradation from the smooth air rating because of either control difficulties or an intolerable indicated acceleration environment.

## Results of the Experiment

### Optimum Longitudinal Gain

#### Side controller characteristics

The torque-displacement characteristics of the elevator axis of the side controller are illustrated by the calibration plot of Fig. 7. The device exhibits a number of nonlinearities such as near-zero spring rate around zero displacement, break-out forces or preload, and hysteresis or friction that was non-uniform.

The optimum longitudinal control gain data will be presented and discussed in the following sections in terms of stick displacement. To present the control gain data in terms of stick force or torque, it would be necessary to linearize the side controller characteristics. Rather than do this, the side controller torque-displacement characteristics are presented separately in Fig. 7; the control gain data will be presented in terms of stick deflection. This should not, however, be interpreted as implying that the pilot flies the airplane by stick deflection rather than stick force.

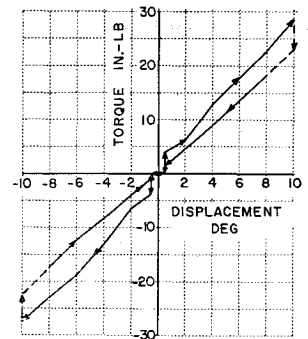
#### Optimum Gains as Function of $n_z/\alpha$

In the analysis of longitudinal gain data, it was found that plotting the steady state  $\alpha/\delta_{ES}$ ,  $n_z/\delta_{ES}$ , and  $\theta/\delta_{ES}$  gains the nondimensional parameter  $L_\alpha V/g$  was the most informative. Assuming constant speed, and neglecting lift due to elevator,  $L_\alpha V/g = n_z/\alpha$ . It should be noted that for a given lift curve slope and wing loading,  $L_\alpha V/g$  is proportional to dynamic pressure.

Plotted in Fig. 8 are the optimum longitudinal control gains selected by pilot A for  $f_n = 0.7$ ,  $\zeta = 0.3$ , pilot B for  $f_n = 0.3$ ,  $\zeta = 0.3$ , and pilot C for  $f_n = 0.7$ ,  $\zeta = 0.7$ . These plots indicate that, for a given short period, the pilots tended to select constant steady state angle of attack gain (or pitch attitude gain at the short period) when  $n_z/\alpha < 10$  g/rad, and to select constant steady state normal acceleration gain when  $n_z/\alpha > 10$  g/rad. The steady state pitch rate gains corresponding to the pilot-selected gear ratios form a family of curves with true speed as a parameter.

The heavy dashed lines in Fig. 8 were obtained by averaging the values of  $(\delta_e/\delta_{ES})(M\delta/\omega_n^2)$  when  $n_z/\alpha < 10$  g/rad and by

Fig. 7 Side controller longitudinal characteristics.



averaging the values of  $(\delta_e/\delta_{ES})(VL_\alpha/g)(M\delta/\omega_n^2)$  when  $n_z/\alpha > 10$  g/rad. The major trends of the optimum longitudinal gain data are thought to be well represented by these dashed lines. It should be noted that the numerical values of the gains selected are applicable only to the particular side controller used in the experiment. If different springs were used in the side controller, or a side controller of different design were used or a center stick or wheel were used, the numerical values of the optimum gains would be different in each case but the functional relation with  $n_z/\alpha$  would be expected to prevail.

Examination of the pilot comments reveals that the optimum longitudinal gains were selected after weighing the often conflicting requirements of good pitch attitude control, adequate  $g$ -limiting protection and satisfactory steady forces in turns. The direction in which each of these factors tended to bias the pilot's choice of longitudinal gain and the range of  $n_z/\alpha$  over which this influence was exerted is shown in the sketch of Fig. 9.

The desire for low steady control forces in turns was present at all values of  $n_z/\alpha$  and always tended to increase the gain selected. This requirement has less influence at low  $n_z/\alpha$  because the bank angle usable was restricted. For  $n_z/\alpha < 10$  g/rad the pilot's desire for precise pitch attitude control without overshoot and oscillation tended to limit the maximum longitudinal control gain that he would accept. For  $n_z/\alpha > 10$  g/rad the maximum longitudinal control gain was limited by the pilot's desire to have precise control of normal acceleration and to be able to rely on control feel to prevent exceeding the acceleration limits of the airframe.

#### Variation With Pilot

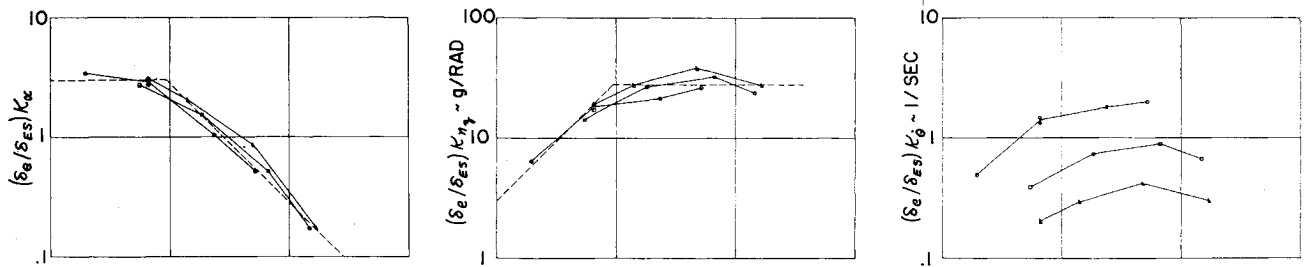
For high  $n_z/\alpha$ , pilot A preferred steady state normal acceleration gain level approximately one-half that preferred by pilot C. Pilot B's preference was intermediate between that of pilots A and C. Thus pilot A liked stick force per  $g$  approximately twice as heavy as pilot C. This result is consistent with past experience with these pilots. Pilot A considered  $g$ -limiting by control feel to be of primary importance and so tended to select heavier control forces. For low  $n_z/\alpha$ , pilots A and B selected essentially the same average gain level for  $f_n = 0.7$ ,  $\zeta = 0.7$ , and  $f_n = 0.3$ ,  $\zeta = 0.3$ . However, pilot C's preference was always appreciably higher than either pilot A's or pilot B's average preferred gain.

#### Pilot Rating Data

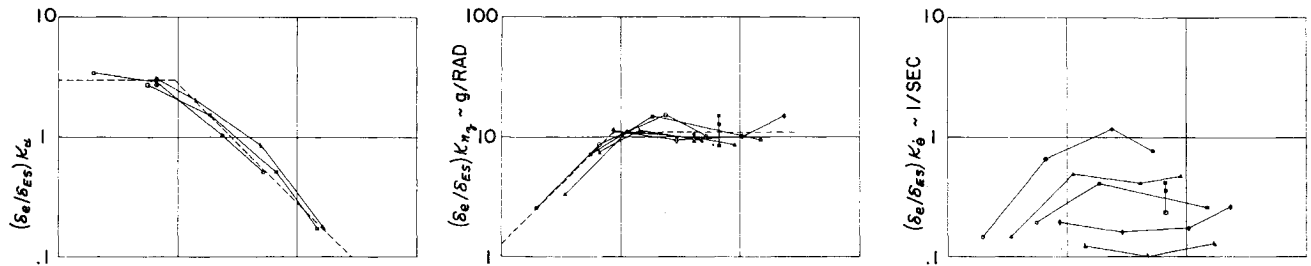
##### Effect of $L_\alpha$ and $V$

The time histories of normal acceleration, angle of attack, pitch attitude, pitch rate and incremental velocity plotted in Fig. 1 are for seven combinations of  $L_\alpha$  and  $V$  for the  $f_n = 0.7$ ,  $\zeta = 0.7$  short period pole. The time histories represent the response of each configuration to a two-degree side controller step input. The stick to elevator gear ratio used for each configuration is based on the averaged longitudinal gain data of pilot B.

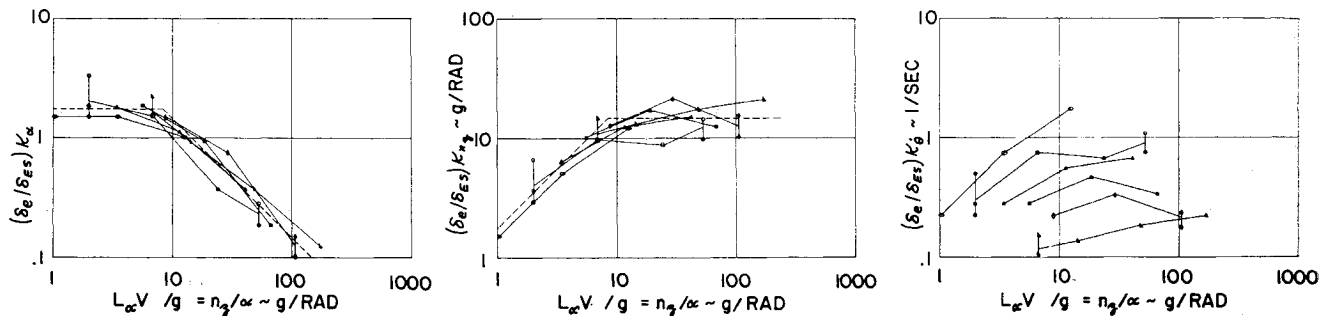
### PILOT C — $f_n = .7$ CPS, $\zeta = .7$



### PILOT A — $f_n = .7$ CPS, $\zeta = .3$



### PILOT B — $f_n = .3$ CPS, $\zeta = .3$



V ~ KNOTS	
○	130
○	250
△	420
△	700
◇	1100
△	1800

Fig. 8 Optimum longitudinal control gains selected by pilots.

As indicated in the Introduction, the individual and combined effects of  $L_\alpha$  and  $V$  on the relative magnitude and phase of the vehicle responses can be seen through detailed examination of the plotted responses in Fig. 1 together with the approximate expressions of Eqs. (8–12).

The relation between normal acceleration and angle of attack expressed by Eq. (11) can be seen by comparing configurations *C*, *D*, and *E* of Fig. 1. For high  $L_\alpha$  and high  $V$  (configuration *C*), the acceleration response is large for a very small angle of attack change, whereas for low  $L_\alpha$  and low  $V$  (configuration *E*), the acceleration response is small for a large angle of attack change. Thus large accelerations result from very small pitch attitude changes when  $n_z/\alpha$  is large (configuration *C*). The large angle of attack changes required to maneuver when  $n_z/\alpha$  is low are accompanied by large drag changes that result in relatively large speed changes. This situation is illustrated by comparing the response of configuration *E* with that of configurations *D* and *C*. The relation between pitch rate and normal acceleration is expressed by Eq. (8), and the relation between pitch rate and angle of attack is expressed by Eq. (9).

In response to an elevator step input, both  $n_z$  and  $\alpha$  make step-like changes. The form of the transient is typically

second order with characteristics determined by the short period dynamics. The pitch rate response, however, starts with an initial slope and leads both  $n_z$  and  $\alpha$  during the short period transient. The time constant of the lead term is proportional to  $1/L_\alpha$ . This aspect of the transient motion is illustrated by the responses of configurations *A*, *B*, and *C*, or *E*, *F*, and *G* of Fig. 1.

The steady state pitch rate resulting from an elevator step is related to the steady state normal acceleration by  $g/V$  as illustrated by the responses of configurations *B*, *D*, and *F*. For a given normal acceleration the higher the velocity, the smaller the steady state pitch rate will be.

The steady state angle of attack corresponding to a given steady pitch rate is proportional to  $1/L_\alpha$ . This relation is illustrated by the responses of configurations *A*, *B*, and *C*. The steady pitch rate is equal for these three configurations and the steady state angle of attack is seen to be inversely proportional to the value of  $L_\alpha$  for each configuration.

Although configurations *A* and *F* have greatly different values of  $V$  and  $L_\alpha$  they have the same value of  $n_z/\alpha$ . The normal acceleration and angle of attack responses of these two configurations are nearly identical; however, the pitch attitude and pitch rate responses are significantly different.

Although the maximum pitch rate which occurs during the transient is nearly equal for the two cases, because of the higher velocity of configuration 4, the steady pitch rate is considerably lower than that of configuration  $F$ .

It should be noted that the maximum pitch rate of configuration  $F$  occurs at a slightly later time than that of configuration  $A$ . In servo terminology, this is because of the phase contribution of the numerator term in the pitch rate to elevator transfer function.

Configuration  $F$  exhibits a larger change in velocity following the step input than does configuration  $A$ . Although the angle of attack and thus, the drag time histories, are the same for these two configurations, because of the larger pitch angle of configuration  $F$ , the gravity component along the longitudinal axis is larger and this causes the larger change in velocity.

The effects of short period frequency and damping ratio on the time histories of normal acceleration, angle of attack, and pitch attitude can be observed by comparing responses in Figs. 1 and 10. The character of the pitch attitude time history for various combinations of  $f_n$ ,  $\zeta$ ,  $L_\alpha$ , and  $V$  is of particular interest since this response is perhaps the primary response displayed to and sensed by the pilot. Also, it has the most complex transfer function in the short period approximations.

When the short period damping ratio is high, as in Fig. 1, the pitch attitude response to an elevator step is comparatively straightforward in that it can be approximated by an initial lag and two ramps of different slope. However, when the short period damping ratio is low, as in Fig. 10, the pitch attitude response appears considerably more nonlinear and exhibits changes in slope, hesitations, and even reversals of direction which complicate the pilot's task of deciding how much control to apply and when it should be applied. This situation is magnified to some extent when the short period frequency is low because of the longer initial delay, and the longer hesitation following the initial rotation or the longer time during which the slope is reversed following the initial rotation.

In the preceding paragraphs the reader's attention was directed to the various aspects of the response of the open-loop or uncontrolled airplane to elevator stick inputs. In the following paragraphs the relative importance of the various aspects of the airplane's responses will be indicated by the pilot ratings and comments obtained.

In Fig. 11, over-all average ratings for each test configuration are listed for each short period pole on grids of  $L_\alpha$  and  $V$ . Logarithmic scales are used to provide uniform spacing of the test points in this figure. These average ratings were obtained by first averaging the ratings for each pilot when more than one were available and then averaging the individual pilot averages. The averaged rating data of Fig. 11 are interpreted by means of shading. Areas of the  $L_\alpha$ - $V$  grid have been shaded in accordance with the three general categories of the rating scale. The shading of areas between data points is based on engineering judgment and detailed familiarity with the pilot comment data. The arrows at the right of the diagrams of Fig. 11 identify the velocities which were done by pilot C. Thus, the average ratings in these rows include the ratings of all three pilots. Superimposed on Fig. 11 are lines of constant  $n_z/\alpha = L_\alpha V/g$ .

The smooth air data of Fig. 11 establish that pilot acceptance of longitudinal flying qualities is a function of the parameter  $n_z/\alpha = L_\alpha V/g$ . In smooth air for all three short period poles, the most satisfactory pilot ratings were obtained for  $5 \text{ g/rad} < n_z/\alpha < 80 \text{ g/rad}$ . Unacceptable ratings were obtained mainly at the extreme combinations of  $L_\alpha$  and  $V$  examined, i.e., at the very high or very low values of  $n_z/\alpha$ .

The unacceptable ratings in these areas are related to the magnitude of the angle of attack change required to maneuver the airplane. When  $n_z/\alpha$  is very low, extremely large changes in angle of attack must be made to develop the incremental lift force required to maneuver the airplane. There are a

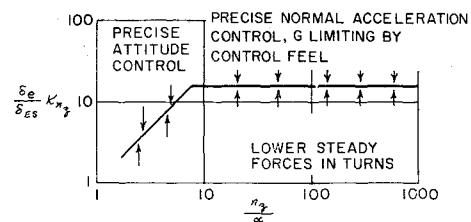


Fig. 9 Factors affecting pilots' choice of longitudinal gain.

number of consequences which result from this large angle of attack change.

1) Large drag changes occur, thus throttle coordination is required if airspeed is to be maintained.

2) For abrupt elevator inputs, the initial pitch rate associated with the rotation about the c.g. may be quite large compared to the steady pitch rate resulting from the flight path curvature.

3) The maximum allowable angle of attack produces only small normal accelerations. Therefore, the bank angle must be restricted to maintain level flight; large attitude changes are required when rolling into and out of turns; and for maneuvers, the airplane must be rotated through a large angle to develop a small normal acceleration; this attitude must be held for considerable time until the small normal acceleration integrates into a flight path change; during this time the pilot tends to lose correspondence between the airplane's pitch attitude and its flight path.

When  $n_z/\alpha$  is very large, extremely small changes in angle of attack will produce very large changes in the lift force. The following consequences result from this situation.

1) The airplane can be flown and maneuvered with fixed throttle since the angle of attack changes required to maneuver are small and thus the drag is nearly constant.

2) Control of rate of climb is quite difficult. Large rates of climb occur for no change in power and very small changes in pitch attitude.

3) Control of normal acceleration is a problem. Caution is required in maneuvers to prevent exceeding the structural limits of the airframe.

For a given value of  $n_z/\alpha$ , the pilots tended to rate the flying qualities of the very high velocity configurations somewhat less satisfactory than they did the intermediate and low velocity configurations. There are several reasons for the higher (less satisfactory) average ratings obtained at the higher velocity.

Consider first the low  $n_z/\alpha$  situation. For a given  $n_z/\alpha$ , the value of  $L_\alpha$  is lower for the high velocity configurations.

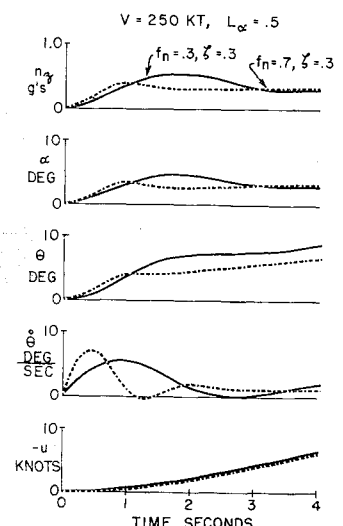


Fig. 10 Longitudinal response to  $2^\circ$  elevator stick step input.

Therefore, the initial pitch rate overshoots the final pitch rate considerably more for the high velocity configurations. That is, the initial pitch response as the airplane rotates about its c.g. may be quite rapid compared to the pitch change resulting from flight path curvature if the short period frequency is high. The longitudinal control gain selected in this region of  $n_z/\alpha$  is a compromise between the initial pitch attitude response and the steady forces in turns with the result that the attitude gain tends to be high. The pilot, therefore, tends to have difficulty in maintaining precise attitude control, and this contributes to the rating degradation.

Another factor that contributes to the rating degradation at high velocity and low  $n_z/\alpha$  is the fact that a given rate of turn requires a larger bank angle when the velocity is high. Thus the bank angle limitation imposed on the configuration by the low  $n_z/\alpha$  becomes more objectionable to the pilot at the high velocity.

Consider next the high  $n_z/\alpha$  situation. When  $n_z/\alpha$  is large, the angle of attack change required to maneuver is small and thus the initial pitch change due to rotation about the c.g. is small. Also, when the velocity is high the steady state pitch rate is small for a given normal acceleration.

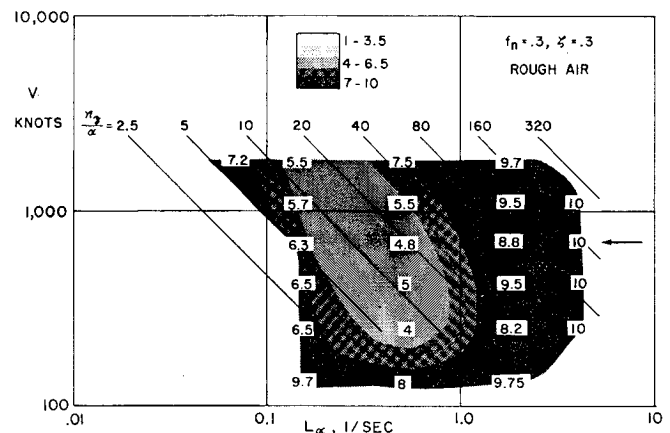
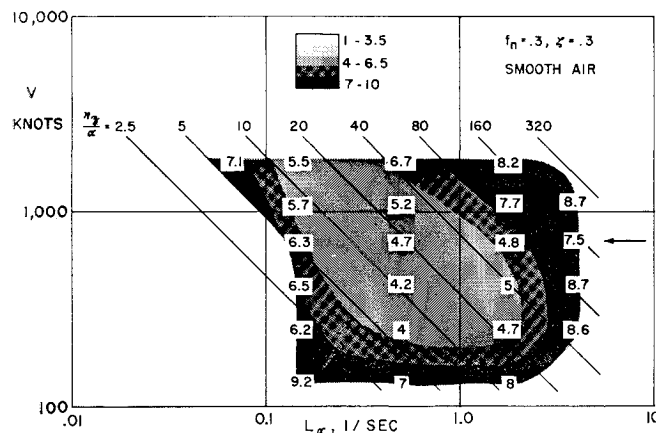
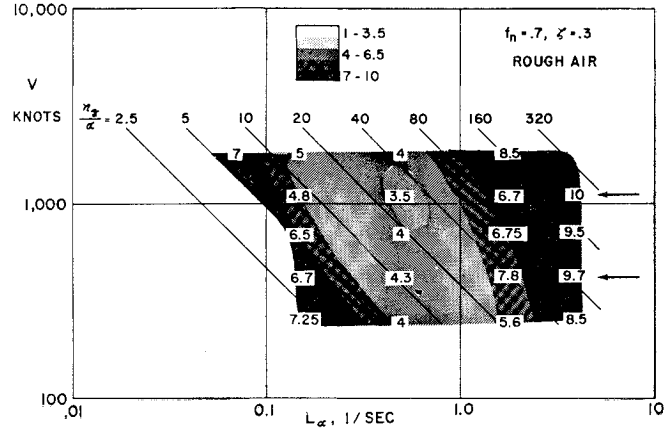
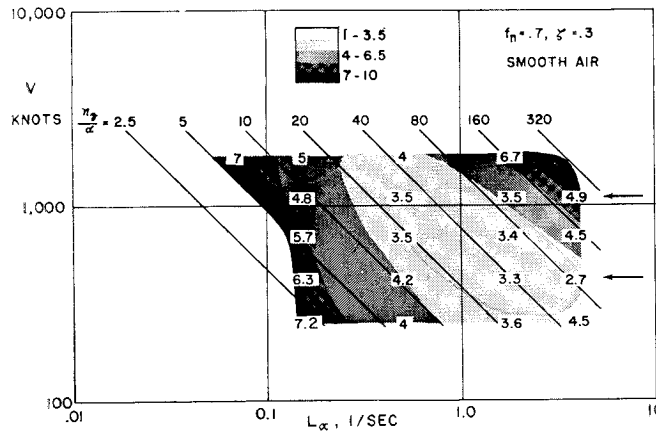
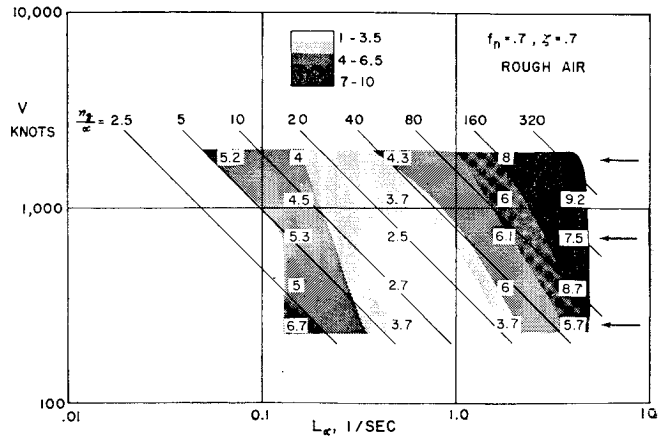
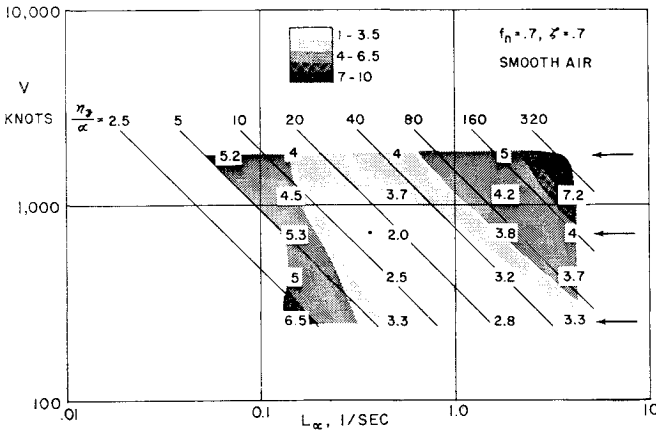


Fig. 11 Averaged pilot rating data.



These two factors combine when  $n_z/\alpha$  and  $V$  are both large. The result is that the airplane can be maneuvered throughout its normal acceleration envelope with very small pitch attitude changes. The pilots found that the attitude indicator was not adequate for the flight control task in this situation. That is, when maneuvering, nearly undetectable changes in the attitude display corresponded to quite large normal acceleration and rate of climb responses. This of course made the flight task more difficult simply because one of the primary instruments that the pilots normally used no longer provided usable information.

#### Effect of short period dynamics

The three short period poles selected for use in the ground simulator program were ranked by the pilots, in the flight tests of Refs. 4-8 in the following order of preference (see Fig. 6):

Good	$f_n = 0.7$	$\zeta = 0.7$
Fair	$f_n = 0.7$	$\zeta = 0.3$
Poor	$f_n = 0.3$	$\zeta = 0.3$

The shaded diagrams of Fig. 11 also indicate this general order of preference. The most satisfactory ratings were obtained for the  $f_n = 0.7$ ,  $\zeta = 0.7$  short period pole, and the least satisfactory ratings were obtained for the  $f_n = 0.3$ ,  $\zeta = 0.3$  short period pole. In all cases the most satisfactory ratings were obtained when  $n_z/\alpha$  was greater than 5 g/rad and less than 80 g/rad. It should be noted that an airplane with good short period dynamics and an extreme value of  $n_z/\alpha$  may be rated less satisfactory than an airplane with poor short period dynamics and a good value of  $n_z/\alpha$ .

#### Effect of rough air

The averaged pilot ratings for rough air are listed on the shaded diagrams of Fig. 11. The degradation in pilot ratings from smooth air to rough air is graphically illustrated by the diagrams of Fig. 11. These experimental results illustrate the relative importance of  $L_\alpha$ ,  $f_n$ ,  $\zeta$ , and  $V$  in determining the response to turbulence. The most severe rating degradations occurred for the highest  $L_\alpha$  values and for the low frequency, low damping ratio configurations. The reduction in normal acceleration response at the c.g., which occurs for higher short period frequency and damping ratio, is reflected in the pilot ratings of Fig. 11. The smallest rating differences were obtained for the short period pole  $f_n = 0.7$ ,  $\zeta = 0.7$ .

For a given value of  $L_\alpha$  the effect of true speed is to shift the power spectrum of the turbulence to higher frequency where the airplane's normal acceleration response is higher. This effect is reflected by the larger rating differences for the higher velocities. The pilots also noted the increase in the frequency of the normal acceleration fluctuations at the higher velocities. The rough air caused essentially no degradation in the pilot rating data when  $L_\alpha < 0.5$ . This result suggests the possibility of providing gust alleviation through servo actuation of flaps or spoilers to reduce  $L_\alpha$ .

In the simulator program the normal acceleration at the c.g., assuming a rigid airframe, was displayed to the pilots. In some cases the pitching motions were noticeably large and the pilots would qualify their ratings by noting that if they were seated far from the c.g., their ratings might be influenced by the normal acceleration caused by angular accelerations.

It should be noted that when an airplane encounters an up-gust for example, it heaves and weathervanes into the gust. The resulting motion for a rigid airframe is such as to diminish the normal acceleration of points ahead of the c.g. and to increase it for points aft of the c.g. If the airframe is elastic, then the predominant structural modes must also be con-

sidered to determine the normal acceleration response at a given location along the fuselage.

#### Conclusions

A fixed-base simulator experiment has been conducted to investigate the effects of true speed and the parameter  $L_\alpha = \rho S V / 2m C_{L_\alpha}$  on the longitudinal flying qualities of piloted aircraft. The major conclusions are given in the following paragraphs.

The longitudinal control gain selected by the pilots was a compromise which weighed the often conflicting requirements for good pitch attitude control, adequate  $g$ -limiting protection, and satisfactory steady forces in turns.

1) For  $n_z/\alpha$  less than approximately 10 g/rad, the pilots tended to choose constant pitch attitude gain at the short period.

2) For  $n_z/\alpha$  greater than approximately 10 g/rad, the pilots tended to choose constant normal acceleration gain.

3) The average longitudinal control gain considered optimum by one pilot differed by a factor of two from that considered optimum by another pilot.

In smooth air, pilot acceptance of longitudinal flying qualities is a function of the parameter  $n_z/\alpha = L_\alpha V/g$ .

1) The most satisfactory ratings were obtained when  $5 < n_z/\alpha < 80$  g/rad.

2) Unacceptable ratings tended to result when  $n_z/\alpha < 5$  g/rad or  $n_z/\alpha > 80$  g/rad. The unacceptable ratings at low  $n_z/\alpha$  were based primarily on the difficulty in maintaining airspeed and the restriction on usable bank angle. The unacceptable ratings at high  $n_z/\alpha$  were based primarily on the difficulty in controlling normal acceleration and rate of climb and on the inadequacy of the pitch attitude indicator as a source of information.

3) The results of previous investigations of the effect of short period dynamics on longitudinal flying qualities must be qualified since these investigations were in general conducted at specific values of  $L_\alpha$  and  $V$  or  $n_z/\alpha$ . If  $n_z/\alpha$  falls within the optimum range, the regions of optimum and acceptable characteristics in terms of short period frequency and damping ratio are broader than otherwise; conversely, for less desirable values of  $n_z/\alpha$  the more demanding the requirements for good values of short period frequency and damping ratio.

The primary factors that determine the normal acceleration response of a rigid airplane to rough air are  $L_\alpha$ ,  $f_n$ ,  $\zeta$ , and  $V$  together with the frequency spectrum characteristics of the rough air.

1) If  $L_\alpha$  is low, the normal acceleration response is small for nearly all values of the other factors. However, if  $L_\alpha$  is high, the acceleration response is lessened by a high short period frequency and a high short period damping ratio.

2) The effect of velocity is to change the frequency spectrum of the turbulence encountered.

The foregoing conclusions must, of course, be qualified by the fact that a fixed-base simulator has been used to simulate flight regimes where normal acceleration and pitching acceleration cues may be significant to the pilot's control function. It is the author's opinion, however, that the general conclusions are sufficiently valid to illustrate that, in specifying longitudinal handling qualities, consideration must be given to task requirements, flight condition, and transfer function numerator factors in addition to short period dynamics.

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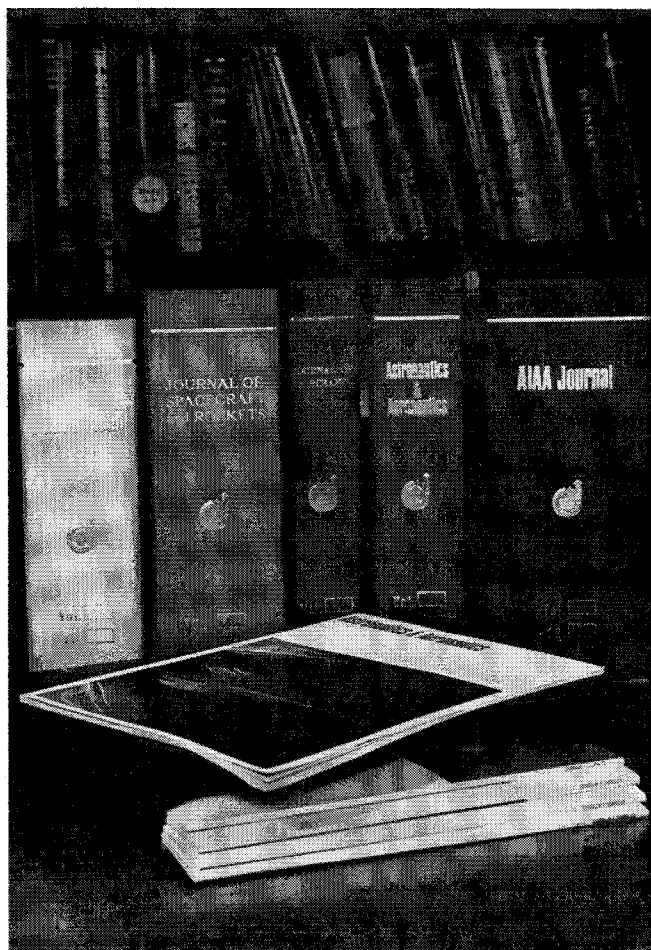
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